Additional program to Coherent Structure Tracking (CST) algorithm: computation of the divergence from spherical velocities (V_φ, V_θ)

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1 Introduction

Once the CST algorithm has computed the Solar surface velocities (e.g V_r, V_φ, V_θ on the $(\vec{x}, \vec{y})$ sky plan. θ and φ are the heliographic latitude and longitude) and their divergences in cartesian coordinates (see the latest user guide from July, 2023), an additional program, written in IDL, will calculate the divergence of the velocity field in spherical coordinates. This divergence, from V_φ and V_θ, are in $1/\mathrm{sec}$ and are comparable to those obtained from helioseismology. The formulas implemented in this program are described in details in section 3.

2 Running the additional program and test case

- Download the package source file div_spher_CST.tgz from MEDOC webpage
- SWIDL software is required. Type the following linux commands:
- tar -xvzf div_spher_CST.tgz
- cd div_spher_CST
- The directory contains input files (latitude_HMI_586.fits, longitude_HMI_586.fits and vr_vtheta_vphi_all_2days_dec2022_deconv.sav) corresponding to the following test case : HMI data (2 days) from December 28 and 29, 2022. These are output files from the CST algorithm (see figure 1 of the latest user guide from July, 2023). More precisely from step 1 for the computing of latitude and longitude, and from step 3 for the computing of velocities V_r, V_φ, V_θ in spherical coordinates on the sky plan or grid $(\vec{x}, \vec{y})$.
- The directory also contains the IDL program (calcul_div_spher_xyz_complet.pro).
- With SWIDL, type the following command : .r calcul_div_spher_xyz_complet.pro
- div_spheric_results.fits contains the results (the divergence of the velocity field in spherical coordinates).
- Figure IDL0.png shows the spherical divergence at each time step.
- Figure IDL1.png shows the spherical divergence (averaged over 16 30-minute images, i.e. 8 hours) calculated from V_φ and V_θ given by CST algorithm (V_x, V_y, V_{Dop}).

3 Formulas implemented in the additional program

Here is a set of variables used in formulas. They are illustrated in figure 1.

- $(\vec{x},\ \vec{y},\ \vec{z})$: Sun reference e.g the reference system in which we observe
- In our case, $V_r = 0$ because the flows are surface flows
- $M(x, y, z)$: a point on solar disk in Sun reference

- θ: heliographic latitude
- $\lambda = \frac{\pi}{2} - \theta$: heliographic colatitude
- φ: heliographic longitude
- V_λ, V_φ: velocity components at point M in Sun reference
- λ and φ depend on (x, y, z)
- R: Solar radius

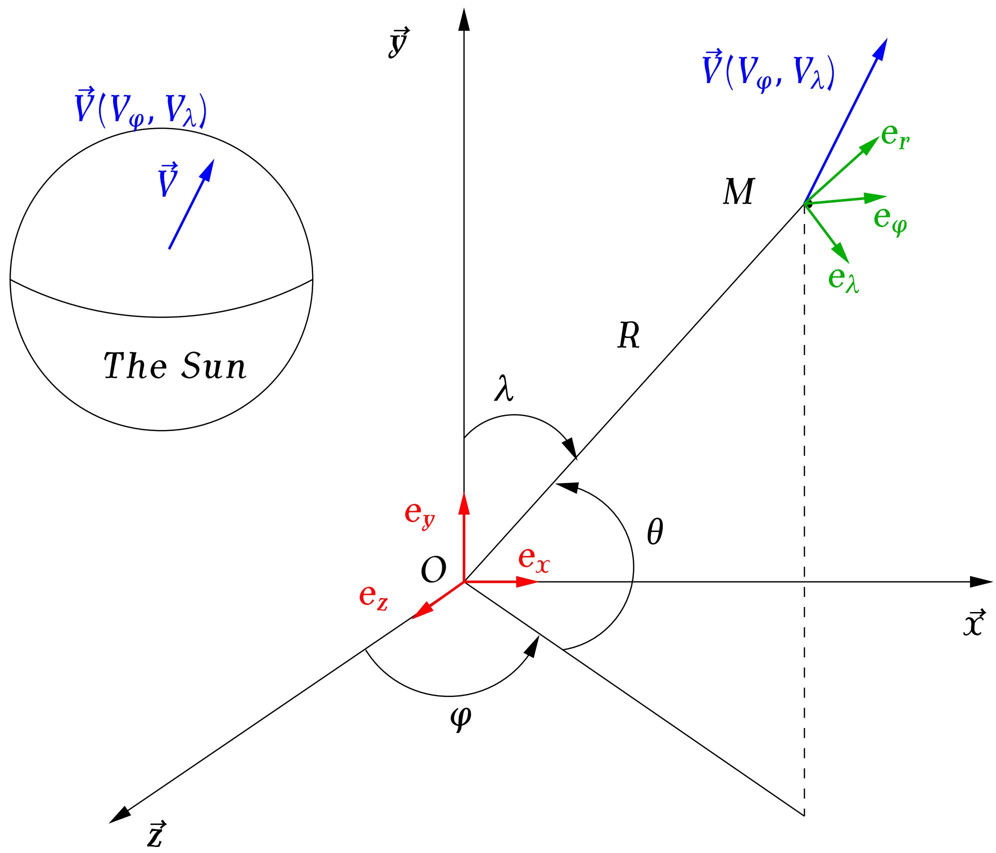

Figure 1: Coordinate systems

The divergence of the velocity field in spherical coordinates is written as:

$$div(\vec{V}) = \frac{1}{R\sin\lambda}\left[\sin\lambda\,\frac{\partial V_\lambda}{\partial\lambda} + \cos\lambda.V_\lambda\right] + \frac{1}{R\sin\lambda}\frac{\partial V_\varphi}{\partial\varphi} \tag{3.1}$$

Let's write the relationship between dx, dy, $d\lambda$, dy.

As $\overrightarrow{OM} = R.\vec{e_r} = R\,(\cos\lambda.\vec{e_y} + \sin\lambda\cos\varphi.\vec{e_z} + \sin\lambda\sin\varphi.\vec{e_x})$, then

$$
\begin{aligned}
d\overrightarrow{OM} &= -R\sin\lambda\ d\lambda.\vec{e_y} + R\ (\cos\lambda\cos\varphi\ d\lambda - \sin\lambda\sin\varphi\ d\varphi)\vec{e_z} \\
&+ \ R\ (\cos\lambda\sin\varphi\ d\lambda + \sin\lambda\cos\varphi\ d\varphi)\vec{e_x} \\
&= dx.\vec{e_x} + dy.\vec{e_y} + dz.\vec{e_z},
\end{aligned}
$$

thus

$$
\begin{cases}
dx = R\ (\cos\lambda\sin\varphi\ d\lambda + \sin\lambda\cos\varphi\ d\varphi) \\
dy = -R\sin\lambda\ d\lambda \\
dz = R\ (\cos\lambda\cos\varphi\ d\lambda - \sin\lambda\sin\varphi\ d\varphi).
\end{cases}
\tag{3.2}
$$

We know that

$$
\begin{cases}
\dfrac{\partial V_\lambda}{\partial \lambda} = \dfrac{\partial V_\lambda}{\partial x}\dfrac{\partial x}{\partial \lambda} + \dfrac{\partial V_\lambda}{\partial y}\dfrac{\partial y}{\partial \lambda} + \dfrac{\partial V_\lambda}{\partial z}\dfrac{\partial z}{\partial \lambda} \\[2ex]
\dfrac{\partial V_\varphi}{\partial \varphi} = \dfrac{\partial V_\varphi}{\partial x}\dfrac{\partial x}{\partial \varphi} + \dfrac{\partial V_\varphi}{\partial y}\dfrac{\partial y}{\partial \varphi} + \dfrac{\partial V_\varphi}{\partial z}\dfrac{\partial z}{\partial \varphi}.
\end{cases}
\tag{3.3}
$$

According to (3.2), we have

$$
\begin{cases}
\dfrac{\partial x}{\partial \lambda} = R\cos\lambda\sin\varphi \\[2ex]
\dfrac{\partial y}{\partial \lambda} = -R\sin\lambda \\[2ex]
\dfrac{\partial z}{\partial \lambda} = R\cos\lambda\cos\varphi \\[2ex]
\dfrac{\partial x}{\partial \varphi} = R\sin\lambda\cos\varphi \\[2ex]
\dfrac{\partial y}{\partial \varphi} = 0 \\[2ex]
\dfrac{\partial z}{\partial \varphi} = -R\sin\lambda\sin\varphi.
\end{cases}
\tag{3.4}
$$

Then (3.3) can be written as

$$
\begin{cases}
\dfrac{\partial V_\lambda}{\partial \lambda} = R\cos\lambda\sin\varphi\,\dfrac{\partial V_\lambda}{\partial x} - R\sin\lambda\,\dfrac{\partial V_\lambda}{\partial y} + R\cos\lambda\cos\varphi\,\dfrac{\partial V_\lambda}{\partial z} \\[2ex]
\dfrac{\partial V_\varphi}{\partial \varphi} = R\sin\lambda\cos\varphi\,\dfrac{\partial V_\varphi}{\partial x} - R\sin\lambda\sin\varphi\,\dfrac{\partial V_\varphi}{\partial z}.
\end{cases}
\tag{3.5}
$$

Thanks to (3.5), the divergence of the velocity field (3.1) can then be written as

$$\begin{aligned} div(\vec{V}) = \cos\lambda\sin\varphi\,\frac{\partial V_\lambda}{\partial x} - \sin\lambda\,\frac{\partial V_\lambda}{\partial y} + \cos\lambda\cos\varphi\,\frac{\partial V_\lambda}{\partial z} \\ + \frac{V_\lambda}{R\tan\lambda} + \cos\varphi\,\frac{\partial V_\varphi}{\partial x} - \sin\varphi\,\frac{\partial V_\varphi}{\partial z} \end{aligned} \tag{3.6}$$

Due to the orientation of angles θ and λ, we have $V_\lambda = -V_\theta$ and $d\theta = -d\lambda$.

Then $\frac{\partial V_\lambda}{\partial x} = -\frac{\partial V_\theta}{\partial x}$, $\frac{\partial V_\lambda}{\partial y} = -\frac{\partial V_\theta}{\partial y}$, $\frac{\partial V_\lambda}{\partial z} = -\frac{\partial V_\theta}{\partial z}$.

The divergence of the velocity field in spherical coordinates can be written as follows

$$\begin{aligned} div(\vec{V}) = -\sin\theta\sin\varphi\,\frac{\partial V_\theta}{\partial x} + \cos\theta\,\frac{\partial V_\theta}{\partial y} - \sin\theta\cos\varphi\,\frac{\partial V_\theta}{\partial z} \\ + \cos\varphi\,\frac{\partial V_\varphi}{\partial x} - \sin\varphi\,\frac{\partial V_\varphi}{\partial z} + \frac{\tan\theta}{R}\,V_\lambda \end{aligned} \tag{3.7}$$

The term $\frac{\tan\theta}{R}\,V_\lambda$ is negligeable because R is large.

Note that the set of equations (3.2) can be written as

$$\begin{cases} \partial x = -R\sin\theta\sin\varphi\;d\theta + R\cos\theta\cos\varphi\;d\varphi \\ \partial y = R\cos\theta d\theta \\ \partial z = -R\,(\sin\theta\cos\varphi\;d\theta + \cos\theta\sin\varphi\;d\varphi) \end{cases} \tag{3.8}$$

which are implemented in the IDL program “calcul_div_spher_xyz_complet.pro".